MATH 244: Discrete Mathematics

Summer 2021

Instructor:	Andrei (Cosmin) Pohoata	**Time:**	MWF 9-11:15am
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Course description: Basic concepts and results in discrete mathematics. Combinatorics is an area of mathematics that deals with finite objects: sets, permutations, relations, partitions, graphs, incidences, etc. These simple combinatorial objects can be used to express beautiful mathematical ideas with little technical work. The goal of this course is to showcase as many such ideas as possible and, along the way, to develop problem-solving techniques that have broad applications in multiple areas in (and out of) mathematics.

The topics that we will explore in this course fall into four broad categories:

- *Counting.* We will enjoy the transparency of combinatorial bijections and the mysterious power of double counting and inclusion-exclusion.
- *Graph theory.* A graph is just a collection of vertices and edges connecting them. But once one starts asking the right questions, a lot of interesting mathematics emerges. In this course, we will study many surprising properties of these objects.
- *Ramsey theory.* The pigeonhole principle and induction are the main tools here. Among their many consequences, Ramsey's theorem is singularly far-reaching: a result about the inevitability of structure in sufficiently large graphs, which paved the way to a remarkable theory.
- *Probabilistic method.* An important discovery of XXth century mathematics is that probability is useful even where order reigns. We will see the manifestation of this great idea in combinatorics; we will use it to construct objects with most unlikely properties.

Rough course plan:

- Meeting #1: course overview, basic counting principles, functions
- Meeting #2: mathematical induction
- Meeting #3: principle of inclusion-exclusion, Bonferroni's inequality
- Meeting #4: introduction to graph theory: BFS
- Meeting #5: bipartite graphs, Euler's theorem and Dirac's theorem.
- Meeting #6: spanning trees: Cayley's theorem and two min-cost spanning tree algorithms
- Meeting #7: planar graphs: Euler's formula and some of its consequences
- Meeting #8: extremal graph theory: Mantel's theorem and Turan's theorem
- Meeting #9: graphs without complete bipartite graphs: Kovari-Sos-Turan
- Meeting #10: some applications of extremal graph theory in other areas
- Meeting #11: introduction to Ramsey theory: variants/generalizations of Ramsey's theorem
- Meeting #12: Schur's theorem and the Happy Ending theorem

- Meeting #13: introduction to the probabilistic method, lower bounds for Ramsey numbers
- Meeting #14: more probabilistic method
- Meeting #15: crossing numbers and the Szemeredi-Trotter theorem

Course format: This will be a lecture course with 125 minute lectures on Zoom on every Monday, Wednesday and Friday (there will be a 10 minute break after one hour). Synchronous attendance is not going formally required, but it will be highly recommended and highly encouraged. Attending the course while it happens is not only the best way to keep up with the material but it is also the only way to qualify for participation credit (see below for more details).

Teaching Assistants: TBD

Office Hours: TBD

Textbook: "Invitation to Discrete Mathematics" (2nd ed.) by Jiri Matousek and Jaroslav Nesetril. Acquiring the book is optional, I will be providing lecture notes and/or slides.

Prerequisites: MATH 115 or equivalent.

Grading Policy: Homework (50%), Take Home Midterm Exam (20%), Take Home Final Exam (30%). I also reserve the right to give some small extra credit for participation at the end of the semester.

Homework: After each class, you will be given one problem which will be due the at the beginning of following class (i.e. the problem you get on Monday is due on Wednesday, the problem you get on Wednesday is due on Friday, and the problem you get on Friday is due on Monday).

- Collaboration on homework problems is generally permitted and encouraged.
- No collaboration of any sort is permitted for the exams. More specific guidelines will be provided when the time comes.
- Late homework or late exam submissions will not be accepted unless accompanied by a note from a Dean.

Academic Honesty: At Yale, academic honesty is taken very seriously. Any deviation from the above policies can result in disciplinary actions (and it is not worth it!).